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Theoretical Glaciology

**Material Science of Ice and the
Mechanics of Glaciers and Ice Sheets**



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